Optimal Taxation in a Stochastic Endogenous Growth Model with Congestion

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Abstract

In this paper the impacts of income and consumption taxes are analyzed within a model of stochastic endogenous growth with congestion. It is shown that the optimal amount of governmental input diminishes with uncertainty and that the optimal financing depends on the relation between the degrees of rivalry and relative risk aversion. Due to the insurance effect associated with the taxation of stochastic income flows, the growth effect of taxation is ambiguous. There is a continuum of optimal tax policies which depends on the assumptions about the governmental budget constraint. The results for a balanced budget are contrasted with the outcomes in the setting with government debt. We demonstrate that in both cases the optimal structure of financing government expenditure not only depends on the degree of rivalry, as in the corresponding deterministic congestion models, but also on the degree of risk aversion.

Zusammenfassung

Das vorliegende Papier untersucht die Auswirkungen von Konsum– und Einkommensteuer im Rahmen eines stochastischen endogenen Wachstumsmodells mit Congestion. Es wird gezeigt, daß die optimale Menge des staatlich bereitgestellten Produktionsfaktors mit steigendem Risiko abnimmt und daß die optimale Finanzierung der staatlichen Ausgaben von der Relation zwischen Rivalitätsgrad und Risikoaversionsgrad abhängt. Aufgrund des akkumulationsfördernden Versicherungseffekts der Besteuerung stochastischer Einkommenskomponenten ist die Wachstumswirkung der Einkommensteuer uneindeutig und es resultiert ein Kontinuum optimaler Finanzierungspolitiken. Die Ergebnisse für den Fall des ausgeglichenen Staatsbudgets werden denjenigen gegenübergestellt, die aus der Erweiterung um Staatsverschuldung als zusätzlich möglichem Finanzierungsinstrument resultieren.

1 Introduction

The analysis of a governmental input in the endogenous growth setting draws back on Barro (1990) and later was adopted by various other authors (see e. g. Barro and Sala-I-Martin (1992), Futagami, Morita, and Shibata (1993), Turnovsky (1995a) as well as Turnovsky and Fisher (1998)). While the assumption of the public input as pure public good is extreme, more recent papers refrain from these restrictions and analyze public production inputs as club goods (e. g. Ott (2000)) or public goods, that are subject to congestion. In the first case, user fees may be levied, in the second case congestion causes a negative externality for the remaining economic agents as rivalry in the use of the public input takes place (see e. g. Atkinson and Stiglitz (1980), Cornes and Sandler (1996)). Barro and Sala-I-Martin (1992), Futagami et al. (1993), Turnovsky (1993, 1995b) as well as Turnovsky and Fisher (1998) argue that nearly all sorts of government expenditure are subject to congestion. This leads to the development of theoretical models of endogenous growth that make allowance for this aspect. The policy implications of these models are that in case of proportional congestion (i.e. maximum rivalry in the use of the public input), a distortionary income tax may internalize the negative external effect that goes along with increasing individual production. On the contrary, in case of no congestion, such a distortionary tax would cause a welfare loss. Correspondingly, the optimal fiscal policy depends on the degree of congestion.

Moreover, the growth effects of fiscal policy depend crucially on the assumption of risk. Stochastic endogenous growth models show that policy recommendations depend on the characteristics of uncertainty. The ambiguous impact of income taxation in a stochastic growth model with linear technology was first discussed by Eaton (1981). Recent contributions of Turnovsky (1993, 1995b), Smith (1996), Corsetti (1997) or Clemens and Soretz (1997) analyze governmental activities in several stochastic endogenous growth settings. Our paper assumes an aggregate income risk that affects the macroeconomic equilibrium in various ways and so is substantial for the efficacy of any tax policy. A risk averse individual will take uncertainty into account within the intertemporal consumption decision. If the agent is sufficiently risk averse there is a motive for precautionary saving as defined by Leland (1968) and Sandmo (1970) which influences the optimal response on income taxation. Furthermore, the insurance argument of Domar and Musgrave (1944) as well as Stiglitz (1969) gains importance. Taxation of returns to risky assets may actually increase the demand for these assets because taxation with full loss offset reduces the volatility of capital returns. Two counteracting growth effects may be derived: A decrease in expected return leads to a decline in capital accumulation whereas a reduction in riskiness may encourage growth.

Our paper links both effects, congestion as well as uncertainty and combines them within a model of endogenous growth. A typical congestion function from the public goods literature is adopted (see e. g. Edwards (1990) or Glomm and Ravikumar (1994)). The degree of congestion plays a central role in deducing the impact of public investment on the rate of accumulation of private capital. Hence, the model considers different modes of financing the provision of the public good and enables the government to levy two forms of revenue: A distortionary income tax and a non–distortionary consumption tax. To clarify the impact of distortionary income taxation on the individual behavior the income tax is split up into one part that is levied on deterministic income components and another part that is levied on stochastic income components. The analysis first takes place within a context of a continuously balanced budget. It is then confronted with the results derived from a setting with government bonds to allow for public debt or surpluses to close the government budget. Independent from the assumptions on the governmental budget constraint it results that the consumption tax amounts to a lump–sum tax thus inducing no growth effects. Besides, concerning the differentiate income tax rate there exists a continuum of of optimal tax policies. But the exact formulation of the continuum is strongly affected by the assumptions on the governmental budget constraint.

The structure of the paper is as follows: After describing the assumptions of the model in section 2, the market equilibrium is analyzed. Part 4 derives the fiscal instruments that allow for optimal financing in case of balanced public sector budget, whereas in section 5 the results are compared with the setting with government bonds. The paper closes with a short summary.

2 The model

The economy is populated by N identical infinitely long living individuals who maximize expected lifetime utility as given by

$$U = \mathrm{E}_0 \left[\int_0^\infty \frac{c^{1-\rho}}{1-\rho} e^{-\beta t} \, dt, \right] \qquad \rho > 0, \quad \rho \neq 1. \tag{1}$$

E_0 denotes the mathematical expectation, conditional on time 0 information, $c(t)$ is time t consumption, $e^{-\beta t}$ represents the discount factor with the instantaneous rate of time preference $\beta > 0$. ρ denotes the measure of relative risk aversion and equals the reciprocal of the intertemporal elasticity of substitution. If the degree of relative risk aversion equals unity instantaneous utility is logarithmic.

The representative firm produces a homogeneous good, y, according to the individual stochastic production function

$$y = \alpha\left(\frac{g}{k}\right) k(dt + \sigma dz), \quad \alpha' > 0, \quad \alpha'' < 0, \quad \sigma > 0. \tag{2}$$

y denotes production per capita, k represents the amount of capital available to the representative firm. α may be interpreted as productivity function and satisfies the Inada–conditions. g reflects the individually available amount of the productive governmental input G, thus allowing for the introduction of congestion effects within the analysis. The composition of g will be explained in detail below. For a constant relation of g/k the production function is linear in capital, thus inducing ongoing growth. The introduction of uncertainty within the analysis takes place with another assumption concerning the production function: In each time increment the production is affected by a Hicks–neutral technological disturbance. The Wiener process $dz(t)$ is a continuous Markov process, i. e. the disturbances are serially uncorrelated and $dz \sim N(0, dt)$. For simplicity, depreciation is neglected, and labor–leisure choice is not considered so that the households supply their labor force inelastically.

The parameter g in the production function represents the part of the public input G which is available for individual production. G is provided by the government and equals total quantity of the public good. Usually, such productive expenditure is identified as being expenditure on infrastructure. Within this model the public expenditure is not necessarily non–rival but, depending on the parameter ε, rivalry may occur. This is expressed by the following congestion function

$$g = G \cdot k^{1-\varepsilon} K^{\varepsilon-1} = G\left(\frac{k}{K}\right)^{1-\varepsilon}, \qquad \varepsilon \in [0,1], \tag{3}$$

where K denotes the aggregate stock of capital. The term $(k/K)^{1-\varepsilon}$ represents a scaling down of the aggregate public good available to the individual due to congestion. The absence of any congestion (no rivalry) is represented by $\varepsilon = 1$, in which case the public input is fully available to the representative agent, $g = G$. In this case, the level of the productivity function α and thus individual output depends on the fraction G/k. The other polar case, $\varepsilon = 0$, corresponds to proportional congestion. This reflects a situation in which total amount of the public input is divided between all N individuals. Hence, the individually available amount of the public input reduces to $g = G/N$. This has consequences for production as productivity in case of arising congestion decreases. An increase of G relative to aggregate capital, K, expands y in (2) for a given amount of individual capital, k. On the other hand, an increase in aggregate capital for given government expenditure lowers the public services available to the individual firms, g/k declines and a reduction of y results.[1] If $0 < \varepsilon < 1$, eq. (3) just represents intermediate cases in which the public input is subject to partial congestion.

To finance the provision of the public input, G, the government levies a tax on consumption ω as well as a proportional income tax. The latter consists of two parts, τ and τ' that are set separately, as in Eaton (1981). This allows for disentangling the effects of taxation of deterministic and random income parts. The public expenditure is financed via the revenues resulting from the differentiate income taxes and a consumption tax. In the sections 3 and 4, the budget is assumed to be balanced in each time increment, whereas in section 5 we allow for public debt and surpluses. The government budget then will be closed by the issuing of bonds.

3 Market Equilibrium and Social Optimum

The representative agent maximizes intertemporal expected utility while taking tax rates as well as the initial values for capital k_0 and the stochastic process z_0 as given. Formally, the optimization problem may be rewritten as

$$\max_{c,k} \quad \mathrm{E}_0 \left[\int_0^\infty \frac{c^{1-\rho}}{1-\rho} e^{-\beta t}\, dt \right] \tag{4}$$

$$\text{s. t.} \quad dk = [(1-\tau)\alpha k - (1+\omega)c]\, dt + (1-\tau')\alpha k \sigma\, dz \quad . \tag{5}$$

[1] To pose the assumption of congestion, instead of the relation between public input and private capital one could alternatively use the relation between public input and private output. Then G would have to rise according to total output Y in order for g/y to remain constant. However, the results would be essentially the same.

Employing Itô's Lemma the stochastic Bellman equation evolves to

$$
\begin{aligned}
\mathcal{B} \;=\; & e^{-\beta t}\frac{c^{1-\rho}}{1-\rho}-\beta e^{-\beta t}J(k)+e^{-\beta t}J'(k)[(1-\tau)\alpha k-(1+\omega)c] \\
& +\frac{1}{2}e^{-\beta t}J''(k)\,\sigma_k^2 \qquad (6)
\end{aligned}
$$

where the value function represents the maximum level of lifetime utility and is assumed to be of the time–separable form $e^{-\beta t}\,J(k(t))$. The variance of capital is given by $\sigma_k^2 = \mathrm{E}[dz_k^2]/dt$.

Maximizing the Bellman equation (6) with regard to c and k leads to the two necessary conditions of the optimization problem

$$
c^{-\rho} = (1+\omega)J'(k) \qquad (7)
$$

$$
\begin{aligned}
& J'(k)\left((1-\tau)\alpha(1-\varepsilon\eta)-\beta\right)+\frac{1}{2}J'''(k)\sigma_k^2 \\
& +J''(k)((1-\tau)\alpha k-(1+\omega)c+(1-\tau')^2\alpha^2(1-\varepsilon\eta)k\sigma^2)=0 \quad . \qquad (8)
\end{aligned}
$$

The parameter η denotes the partial production elasticity of the public input G and is given by

$$
\eta \equiv \frac{\partial\alpha}{\partial G}\frac{G}{\alpha} = \frac{\alpha'}{\alpha}\frac{g}{k}. \qquad (9)
$$

Furthermore, the marginal product of capital as perceived by the individuals is given by

$$
\frac{\partial y}{\partial k} = \left(\alpha-\varepsilon\alpha'\frac{g}{k}\right)(dt+\sigma dz) \qquad (10)
$$

and is influenced by congestion as well as by risk. If congestion arises the individually perceived marginal product is suboptimally high may induce excessive growth. The policy implications will be discussed below.

Equation (7) comprises the usual result of intertemporal optimization that marginal utility of consumption corresponds to the (weighted) first derivative of the value function and is equalized across time. It determines the accumulation process together with (8). The optimal time paths for consumption and capital depend on the derivatives of the value function and form a stochastic differential equation in $J(k)$. Hence, a maximum of utility as given by the integral (1) is obtained by determining a function $J(k)$ that solves the first–order conditions.

For expected utility to be bounded, feasible intertemporal consumption paths require the following transversality condition to be satisfied[2]

$$\lim_{t\to\infty} \mathrm{E}\left[e^{-\beta t}J(k)\right] = 0 \quad . \tag{11}$$

Below, the competitive equilibrium allocation depending on certain policy parameters is described. Since the households have constant relative risk aversion and all parameters of the model are time–invariant it is supposed that capital and consumption grow at a common rate (see Merton (1971)). In this case, the propensity to consume out of capital, μ, will be constant in macroeconomic equilibrium

$$c(t) = \mu k(t) \quad . \tag{12}$$

With this it is possible to derive a closed–form solution for optimal consumption

$$\begin{aligned}(1+\omega)\,\mu \;&=\; \frac{\beta}{\rho} + (1-\tau)\alpha - \frac{1}{\rho}(1-\tau)\alpha(1-\varepsilon\eta) \\ &\quad + (1-\tau')^2\alpha^2\sigma^2\left(\frac{1-\rho}{2} - \varepsilon\eta\right) \quad .\end{aligned} \tag{13}$$

Public revenues out of income and consumption tax grow with the same expected rate as capital and consumption. Furthermore, as all N individuals are identical, aggregate capital, K, is equal to Nk. Hence, in equilibrium the ratio g/k in individual production that determines a unique level of α as well as of α' also remains constant. Thus, the production elasticity of governmental input, η, is constant as well. From this follows immediately, that the propensity to consume out of capital is time–invariant and depends on the underlying parameters as well as on the fiscal instruments.

The expected growth rate of the economy, φ, can be obtained from the individual budget constraint (5) together with equation (13)

$$\varphi \equiv \frac{\mathrm{E}[dk]}{k\,dt} = \frac{1}{\rho}\left((1-\tau)\alpha(1-\varepsilon\eta) - \beta\right) + (1-\tau')^2\alpha^2\sigma^2\left(\varepsilon\eta - \frac{1-\rho}{2}\right) \quad . \tag{14}$$

This growth rate is determined endogenously and depends on all parameters that affect aggregate savings. Besides, it contains two components: the first corresponds to the growth rate of the deterministic congestion model while the second part reflects the agent's optimal response to technological risk.

[2]Merton (1969) shows that with linear technology the transversality condition is equivalent with the condition of a positive relation between consumption and wealth.

The specific feature of a congestion model is that investment in the private capital stock not only increases output but results in a second effect. It reduces the ratio of available government expenditure to individual capital, g/k, and hence the productivity measured by α declines as long as G does not increase at the same rate as private capital. This represents a negative external effect of capital accumulation that drives a wedge between private and social return on capital.

The expected growth rate (14) exceeds deterministic growth if the second part of the sum is positive. Concerning the second term of the growth rate it becomes obvious that it is influenced by the underlying technological risk as well as by congestion. But while the effect of arising congestion (diminishing ε) is an unequivocal reduction of expected growth, the influence of risk is ambiguous: For sufficiently risk averse agents, ($\rho > 1 - 2\varepsilon\eta$), the expected growth rate exceeds the one in the deterministic setting. The contrary applies if the agent has too low a motive for precautionary saving, ($\rho < 1 - 2\varepsilon\eta$). Besides, the tax on permanent capital income τ as well as the tax on transitory capital returns τ' affect expected growth. A detailed discussion of the impact of uncertainty and of fiscal instruments follows in the next section.

Due to the properties of the productivity shock, capital is lognormally distributed and follows a random walk with positive drift. Starting from initial capital k_0 at time 0, capital at time t evolves corresponding to[3]

$$k(t) = k_0 \, e^{\left(\varphi - \frac{1}{2}\alpha^2\sigma^2\right) t + \alpha\sigma [z(t) - z_0]} \quad . \tag{15}$$

Consider now a benevolent social planner who maximizes welfare while taking the negative externality of capital accumulation arising from congestion into account. The congestion function in this case is obtained by setting $K = Nk$ in equation (3) and reduces to

$$g = GN^{\varepsilon - 1} \quad . \tag{16}$$

Additionally to consumption and capital, the benevolent social planner decides about the optimal amount of government expenditure. The according necessary condition results in

$$\alpha'(g/k)^* = \frac{N^{-\varepsilon}}{1 - \rho\alpha\sigma^2} \tag{17}$$

[3]This may be derived in analogy to Malliaris and Brock (1982).

which determines the optimal relation g/k by equating marginal product and marginal cost of government expenditure. Together with equation (9) and the congestion function of the planner (16) the optimal elasticity η^* may be derived as

$$\eta^* = \frac{G}{(1-\rho\alpha\sigma^2)N\bar{y}} \tag{18}$$

with $\bar{y} = E[y]/dt$. It is influenced by congestion via individual production. The denominators in equations (17) and (18) are of the same sign as the certainty equivalent of the stochastic capital return.[4] To ensure feasible solutions, the certainty equivalent has to be positive, because the marginal productivity α' is assumed to be positive. Due to the assumption of diminishing returns, $\alpha'' < 0$, the underlying productivity shock reduces the optimal ratio $(g/k)^*$ compared to a deterministic setting.[5] Under uncertainty, the governmental provision of the public input has two impacts: It enhances not only expected capital productivity but additionally increases the volatility of capital return given by σ_k^2. As a consequence, in a society of risk averse agents, the optimal level of government expenditure decreases with a rise in uncertainty. As long as the certainty equivalent of capital return is positive, the second (negative) effect of a rise in government expenditure doesn't overcompensate the first (positive) effect.

Optimization of the social planner leads to the macroeconomic equilibrium determined by

$$\begin{aligned} \mu^* &= \frac{\beta}{\rho} + \frac{\rho-1}{\rho}\alpha + (\rho-1)\alpha^2\sigma^2\left(\eta^* - \frac{1}{2}\right) && (19) \\ \varphi^* &= \frac{1}{\rho}\left(\alpha(1-\eta^*) - \beta\right) + \alpha^2\sigma^2\left(\eta^* - \frac{1-\rho}{2}\right) \quad . && (20) \end{aligned}$$

Equations (18), (19) and (20) determine the social optimum. In the following sections it will be shown that it is possible to attain the social optimum within a competitive economy by the choice of an optimal policy mix. To evaluate alternative fiscal policies the command optimum will serve as reference.

[4]The certainty equivalent is $\alpha - \rho\alpha^2\sigma^2$ for the planner solution. For the determination see e. g. Merton (1992, p. 45).

[5]In the deterministic setting the optimal elasticity is determined by $\alpha'^* = N^{-\varepsilon}$.

4 Fiscal Policy with Balanced Budget

In this section we determine the impact of fiscal policy on expected growth with the assumption of a balanced government budget. Therefore, the macroeconomic equilibrium described in the previous section serves as a base for comparative dynamic analysis. The expected growth rate (14) reflects the optimal response of the representative agent to a given fiscal policy. We now focus on the growth effects of changes in the differentiate income taxes as well as the consumption tax. It results that in the context considered here the effects of fiscal policy on equilibrium growth are ambiguous and do not only depend on the degree of congestion but also on the individual attitude towards risk.

Since it is assumed that utility depends only on consumption, there is no labor–leisure choice, and labor is supplied inelastically. For this reason there are no transitory dynamics in the underlying growth model. Additionally, we assume government to change tax policies exclusively at time $t = 0$. Hence, the consequences of alternative tax policies can be shown by direct comparison between different steady–states.

Concerning the consumption tax rate ω, the well–known result of intertemporal optimization may be derived: a rise of ω lowers the equilibrium level of the propensity to consume (see eq. (13)) whereas the consumption tax is neutral to accumulation. This can be seen as ω does not appear in the solution for expected growth rate (14). As labor is supplied inelastically, the consumption tax has no impact on optimal savings and acts like a lump–sum tax.

We now focus on the effects of changes in the differentiate income tax rates. An increase in the tax on deterministic income parts induces a reduction in expected growth

$$\frac{\partial \varphi}{\partial \tau} = -\frac{\alpha}{\rho}(1-\varepsilon\eta) < 0 \quad . \tag{21}$$

Since taxation of permanent income reduces the expected marginal productivity of capital, accumulation of physical capital becomes less attractive. The level of optimal savings decreases and leads to a decline in equilibrium growth. This effect is reinforced if congestion arises.[6] This negative growth effect reflects the distortionary impact of income taxation known from the deterministic setting.

[6] Note that $\alpha(1-\varepsilon\eta)$ has to be positive for feasible solutions since it represents the expected private marginal return on capital.

In the stochastic environment considered here, a tax on capital returns affects the mean as well as the volatility of capital income streams. If we analyze a rise in the tax on random income parts ambiguous growth effects can be derived

$$\begin{aligned} \frac{\partial\varphi}{\partial\tau'} &= 2(1-\tau')\alpha^2\sigma^2\left(\frac{1-\rho}{2}-\varepsilon\eta\right) \gtrless 0 \\ &\Longleftrightarrow \rho \lesseqgtr 1-2\varepsilon\eta \quad . \end{aligned} \tag{22}$$

According to Leland (1968) and Sandmo (1970), precautionary savings are defined as additional savings due to the uncertainty of future income flows. The riskiness of future income flows leads to two contrary effects on individual savings. Rothschild and Stiglitz (1971, p. 69) describe this ambiguity as "[...] increased uncertainty in the return on savings will either lower savings because 'a bird in the hand is worth two in the bush' or raise it because a risk–averse individual, in order to insure his minimum standard of living, saves more in the face of increased uncertainty". First, an increase in the volatility of future capital returns causes a negative substitution effect on savings. To avoid future uncertainty, momentary consumption is increased at the cost of accumulation. Second, an income stream with higher volatility is associated with lower expected utility. This induces a positive income effect on optimal accumulation to compensate for the loss of future utility. Savings are increased to equalize expected marginal utility over time. Individual optimization leads to precautionary saving if the degree of risk aversion is sufficiently high. The positive income effect then offsets the negative substitution effect.

Taxation of stochastic income components reduces the volatility of capital return while expected return remains unchanged. Whether the representative agent responds to decreasing uncertainty with a decline in precautionary savings or with a rise in capital accumulation crucially depends on the relation between relative risk aversion and the degree of rivalry. If the individual is sufficiently risk averse ($\rho > 1-2\varepsilon\eta$), the reduction in uncertainty due to taxation of stochastic income will lead to a decline of savings out of the precautionary motive. The negative growth effect of the tax on uncertain income parts in this case reinforces the growth diminishing impact of the tax on deterministic income components. If instead the risk aversion is sufficiently low ($\rho < 1-2\varepsilon\eta$), the substitution effect dominates and lower risk due to an increase in τ' induces a switch towards higher capital accumulation. Hence, in this case taxation of stochastic income components increases the resulting growth rate and leads to counter working growth effects of overall income taxation.

Furthermore, the growth effect of a rise in the tax rate on stochastic income parts depends on the degree of congestion, ε. The parameter of rivalry specifies the division of uncertainty into capital risk and income risk.[7] With a lower ε, the agents assign a greater part of risk to the uncertain capital return. The case where the negative external effect is maximal, that is $\varepsilon = 0$, corresponds to the situation of pure capital risk. Individuals have a motive for precautionary saving with $\rho > 1$, as Sandmo (1970) derived. On the contrary, if the government input is a pure public good, that is $\varepsilon = 1$, the individuals perceive only a small part of the uncertainty as capital risk. The remaining uncertainty appears as income risk and induces a decrease in the critical level of risk aversion which is necessary to display a motive for precautionary savings. This leads to an enlargement of the parameter interval in which the motive for precautionary saving is relevant.

An optimal tax policy will eliminate any differences between equilibrium growth (14) and optimal growth (20). Due to congestion, the private marginal product of physical capital depends on the degree of rivalry of government expenditure. As long as $0 < \varepsilon < 1$, the rivalry is neither perfect nor absent and the private capital return is higher than the social one. Within individual optimization, the representative agent does not take into account his contribution to the aggregate capital stock. Thus, the reduction of the ratio between available government expenditure and individual capital is underestimated and the incentive for capital accumulation is suboptimally high. In the case of a proportional congestion ($\varepsilon = 0$), the deviation is maximal, whereas with a pure public good ($\varepsilon = 1$) private and social marginal product of capital coincide.

In opposite to the results in the deterministic congestion model, the desired reduction of the growth rate cannot only be achieved by a positive tax rate on expected income, but also by a taxation of stochastic income parts or a mixture of both instruments. With regard to the latter tax parameter, the optimal policy depends on the degree of risk aversion of the representative individual (see equation (22)). If the agent is sufficiently risk averse, i. e. if there is a motive for precautionary saving, the decrease in expected growth can be induced by a positive tax rate on stochastic income components. On the contrary, if risk aversion is sufficiently low, a subsidy on random income parts leads to the desired decline in expected growth.

Equating the expected growth rates of the decentralized equilibrium (14) and of the social

[7] This distinction draws back on Sandmo (1970) and was applied to stochastic endogenous growth models in Clemens and Soretz (1999).

optimum (20) leads to the following quadratic optimum condition between the income tax parameters

$$(1-\tau)(1-\varepsilon\eta^*) \stackrel{!}{=} (1-\tau')^2\rho\alpha\sigma^2\left(\frac{1-\rho}{2}-\varepsilon\eta^*\right) \\ +(1-\eta^*)-\rho\alpha\sigma^2\left(\frac{1-\rho}{2}-\eta^*\right) \tag{23}$$

where η^* is evaluated at the optimal level of government expenditure as defined in equation (18). The optimal level of the consumption tax, ω^*, is than determined residually together with the budget constraint of the social planner. The relation between the two tax parameters can be expressed as

$$\frac{d\tau}{d\tau'} = \frac{2(1-\tau')\rho\alpha\sigma^2\left(\frac{1-\rho}{2}-\varepsilon\eta^*\right)}{(1-\varepsilon\eta^*)} \quad . \tag{24}$$

The direction of this relation again depends on risk aversion ρ as well as on the degree of congestion ε

$$\frac{d\tau}{d\tau'} \gtrless 0 \quad \Longleftrightarrow \quad \rho \lessgtr 1-2\varepsilon\eta^* \quad \Longleftrightarrow \quad \varepsilon \gtrless \frac{1-\rho}{2\eta^*} \quad . \tag{25}$$

With sufficiently risk averse agents, there is a motive for precautionary saving. An increase in taxation of uncertain income reduces the volatility of capital return thereby decreasing the savings out of precautionary motives and long–run growth. This negative growth effect of a tax on stochastic income parts reinforces the growth diminishing impact of a tax on deterministic income components. Hence, a rise in the tax rate on deterministic income components τ — accompanied by a negative growth effect — can be offset by a decrease in the tax rate on stochastic income parts τ'. In case of a sufficiently low degree of risk aversion the argument reverses. The level of congestion influences the critical value of ρ that separates these two cases as already was explained.

Due to the differentiation of the tax rates, there exists a continuum of optimal tax policies which is shown in figure 1. The curve on the left hand side captures the case with relatively small risk aversion ($\rho < 1-2\varepsilon\eta$), whereas the second subfigure reflects the case of precautionary savings[8] ($\rho > 1-2\varepsilon\eta$).

[8] In this case, the position of the curve may be partly or entirely below the τ'-axes, depending on the value of ε.

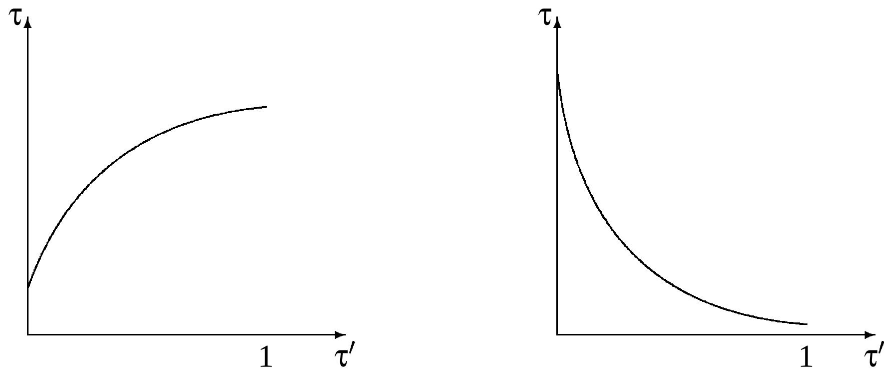

Figure 1: Optimal Fiscal Policy for small versus high relative risk aversion

We now discuss the implications of the relation between the two income tax rates given in equation (23) for the two benchmark cases $\varepsilon = 0$ and $\varepsilon = 1$. As already mentioned, in the case of a pure public good ($\varepsilon = 1$) there exists no external effect. Thus, any optimal tax policy has to remain expected growth unchanged. In this case, the relation between the optimal tax rates evolves to

$$\varepsilon = 1 \quad \Longrightarrow \quad \tau(1-\eta^*) \overset{!}{=} \tau'(2-\tau')\rho\alpha\sigma^2\left(\frac{1-\rho}{2} - \eta^*\right) \quad . \tag{26}$$

Any taxation of deterministic income components must be offset by an appropriate tax rate on stochastic income parts (and vice versa) to ensure optimal growth. If one of the income taxes is neglected, the other tax rate must be set zero. This outcome corresponds to the deterministic setting: since there only exists one income tax parameter, in the case of a pure public good the income tax vanishes and is replaced by a consumption tax (see e. g. Turnovsky (1995a)).

On the contrary, in the case of a proportionally congested governmental input ($\varepsilon = 0$), the optimality condition of an income tax can be described as follows

$$\varepsilon = 0 \quad \Longrightarrow \quad \tau \overset{!}{=} \tau'(2-\tau')\rho\alpha\sigma^2\frac{1-\rho}{2} + \eta^*(1-\rho\alpha\sigma^2) \quad . \tag{27}$$

With the optimal level of g/k which determines the elasticity of governmental input, η^*, given in (18), this condition can be rewritten as

$$\tau \overset{!}{=} \tau'\left(1-\frac{\tau'}{2}\right)\rho\alpha\sigma^2(1-\rho) + \frac{G}{N\bar{y}} \quad . \tag{28}$$

If the income tax on uncertain income is positive, the level of the tax rate τ depends on the degree of risk aversion. Since the governmental input is characterized by proportional congestion, there is a motive for precautionary savings if relative risk aversion exceeds unity (see equation (22)). A tax on stochastic income then reduces savings. Hence, the difference between socially optimal and equilibrium expected growth decreases and the optimal tax rate on deterministic income declines with a increasing risk. On the contrary, if relative risk aversion is less than unity, the opposite argument applies and the optimal tax rate on expected income increases with uncertainty.

In contrast to the deterministic congestion model the optimal income tax rate τ may exceed the expenditure ratio if the tax rate τ' is positive and risk aversion is less than unity. Optimal financing in this case implies a subsidy on consumption to balance the governmental budget. A uniform tax rate, that is $\tau = \tau'$, does only imply a full financing through income taxation if utility is logarithmic ($\rho = 1$). Insofar, it is not possibly to apply the outcomes of the corresponding deterministic setting to uncertainty. The optimal uniform income tax rate is overestimated by the deterministic model if relative risk aversion is greater than unity and underestimated in the opposite case, $\rho < 1$.

If instead the tax rate on stochastic income components is zero, e. g. because stochastic income parts cannot be observed by the government, public expenditure is fully financed by the income tax on deterministic income parts. The income taxation reduces private capital returns and discourages growth. Thus, the negative external effect of capital accumulation is internalized. Within this setting, the consumption tax vanishes because the governmental revenue out of income taxation is sufficient to cover the costs for the provision of the public input.

5 Fiscal Policy with Government debt

In this section, we relax the assumption about the balanced government budget. Instead, the government now may finance deficits by issuing bonds. The value of government bonds is not necessarily positive, that is, government may be a net creditor. Since public revenues out of income tax are stochastic whereas public expenditure is deterministic, the value $b(t)$ of government bonds is stochastic.

The value of bonds is measured in units of output and they are characterized as perpetuities paying an uncertain real return. The expected rate of return is given by i, and the stochastic process of bond return is dz_b. Thus, the value of government bonds evolves according to

$$db = \left[\frac{G}{N} + ib - \tau\alpha k - \omega c\right] dt + b\, dz_b - \tau'\alpha k\sigma\, dz \quad . \tag{29}$$

With these assumptions, there is an additional portfolio decision. The households have to decide which parts of individual wealth, w, they want to invest in physical capital and in government bonds. We assume that wealth is entirely distributed into the two assets. Thus, with the portfolio share of physical capital denoted by n, the portfolio share of government bonds is $1-n$.

As individual wealth is the sum of the two assets, the wealth constraint results in

$$dw = [(1-\tau)\alpha nw + i(1-n)w - (1+\omega)c]\, dt + (1-\tau')\alpha nw\sigma\, dz + (1-n)w\, dz_b \tag{30}$$

and the optimization problem is to maximize expected utility (1) subject to the wealth constraint (30). The Bellman equation now is

$$\begin{aligned} \mathcal{B} \;=\; & e^{-\beta t}\frac{c^{1-\rho}}{1-\rho} - \beta e^{-\beta t}J(w) + e^{-\beta t}J'(w)\,[(1-\tau)\alpha nw + i(1-n)w - (1+\omega)c] \\ & + \frac{1}{2}\, e^{-\beta t}J''(w)\,\sigma_w^2 \end{aligned} \tag{31}$$

with σ_w denoting the standard deviation of wealth. Intertemporal utility is maximized with respect to consumption, wealth and the portfolio share of physical capital, resulting in the necessary conditions

$$c^{-\rho} = (1+\omega)J'(w) \tag{32}$$

$$\begin{aligned} & J'(w)\,((1-\tau)\alpha n(1-\varepsilon\eta) + (1-n)i - \beta) + \frac{1}{2}J'''(w)\sigma_w^2 \\ & + J''(w)\left((1-\tau)\alpha nw + i(1-n)w - (1+\omega)c + \frac{\partial\sigma_w^2}{\partial w}\right) = 0 \end{aligned} \tag{33}$$

$$J'(w)\,[(1-\tau)\alpha(1-\varepsilon\eta)w - iw] + J''(w)\frac{\partial\sigma_w^2}{\partial n} = 0 \quad . \tag{34}$$

Optimal consumption (32) together with the conjecture of a constant propensity to consume out of capital (12) lead to specific functions for the derivatives of the value function. Additionally, we suppose that in steady–state the portfolio shares are constant. That is, all assets grow with the same expected rate and furthermore the evolution of the stochastic processes is equal. Hence, the equilibrium stochastic process of the real return of government bonds is

$$dz_b = \frac{\alpha\sigma}{1-n}(1-(1-\tau')n)dz \quad . \tag{35}$$

Together with the arbitrage condition (34) for optimal portfolio choice this implies the steady–state value of expected real return on bonds[9]

$$i = (1-\tau)\alpha(1-\varepsilon\eta) - \frac{\rho\alpha^2\sigma^2}{1-n}((1-\tau')(1-(1-n)\varepsilon\eta)-1) \quad . \tag{36}$$

Using these informations in the necessary condition (33) leads to the ratio between consumption and capital, μn, is given by

$$\begin{aligned}(1+\omega)\,\mu n \;&=\; \frac{\beta}{\rho} + \frac{\rho-1}{\rho}(1-\tau)\alpha(1-\varepsilon\eta) + (1-\tau)\alpha\varepsilon\eta n \\ &\quad +\alpha^2\sigma^2\left((1-\tau')(1-\varepsilon\eta-\rho(1-(1-n)\varepsilon\eta)) + \frac{\rho-1}{2}\right) \quad .\end{aligned} \tag{37}$$

and is constant in steady–state since the production elasticity η as well as portfolio choice are constant. Now it is possible to determine expected growth out of equation (30)

$$\varphi \equiv \frac{\mathrm{E}[dw]}{w\,dt} = \frac{1}{\rho}\left((1-\tau)\alpha(1-\varepsilon\eta)-\beta\right) + \alpha^2\sigma^2\left(\frac{\rho+1}{2} - (1-\tau')(1-\varepsilon\eta)\right) \tag{38}$$

and the resulting growth effects of tax policy are

$$\frac{\partial\varphi}{\partial\tau} \;=\; -\frac{\alpha}{\rho}(1-\varepsilon\eta) < 0 \tag{39}$$

$$\frac{\partial\varphi}{\partial\tau'} \;=\; \alpha^2\sigma^2(1-\varepsilon\eta) > 0 \quad . \tag{40}$$

The taxation of deterministic income components leads to the growth diminishing effect described in the last section. This outcome is independent of the assumption about the government budget constraint and reflects the reduction in net capital return.

A rise in the tax on stochastic income components reduces the volatility of the net return on capital while the expected return on capital remains constant. Thus, taxation

[9] This outcome only applies if $n \neq 1$. In the case $n = 1$, the value of government bonds is zero, $b = 0$. This implies no taxation of stochastic capital returns, $\tau' = 0$, and no stochastic process of return on bonds, $dz_b = 0$. With this setting, the expected real return on bonds is $i = (1-\tau)\alpha(1-\varepsilon\eta) - \rho\alpha^2\sigma^2(1-\varepsilon\eta)$.

of stochastic capital returns induces the reverse of a mean preserving spread as defined by Rothschild and Stiglitz (1970). The risk associated with the after tax return on capital decreases and capital accumulation gets more attractive for a risk averse agent. This insurance argument was first discussed by Domar and Musgrave (1944) and further developed by Stiglitz (1969). Since taxation with full loss–offset reduces the variance of returns, it may increase the demand in risky assets. Hence, with the assumption of government bonds which balance the government budget constraint, the growth effect of the tax on stochastic income components changes substantially. It does not longer depend on the motive for precautionary saving, but is now unambiguously positive.[10]

With these results we now can determine optimal fiscal policies. For a welfare maximizing policy, the equilibrium growth rate has to equalize optimal growth as determined in equation (20). This implies the linear relationship

$$\tau = \rho\alpha\sigma^2\tau' + \frac{\eta^*(1-\varepsilon)}{1-\varepsilon\eta^*}(1-\rho\alpha\sigma^2) \tag{41}$$

between the tax rates on deterministic and stochastic income components. The production elasticity η^* is evaluated at the optimal level of government expenditure as given by equation (18). As in the last section with balanced government budget there results a continuum of optimal tax policies. But the relation between the optimal tax rates now is unambiguously positive. This reflects the unambiguously positive growth effect of the taxation of stochastic income components. Starting from an optimal tax policy, a rise in the tax rate on deterministic income leads to a decline in expected growth which only can be compensated by a the positive growth effect of an increase in the tax rate on stochastic income.

Condition (41) for the optimality of fiscal policy is shown in figure 2. The two parallel lines show the special cases of a pure public and a pure private good. The remaining cases ($0 < \varepsilon < 1$) are found in between. For feasible solutions the slope is less than unity as can be seen from equation (17). The third line ($\tau = \tau'$) reflects the case of uniform taxation of deterministic and stochastic income parts.

[10] This difference in the outcomes is already known from other stochastic endogenous growth models with taxation. Independently from the assumptions about the production function, the growth effects are ambiguous as long as the public budget is continuously balanced (see e. g. Smith (1996)) and get unambiguously positive with the assumption of government bonds (see e. g. Turnovsky (1995a, chapter 14), Clemens and Soretz (1997)).

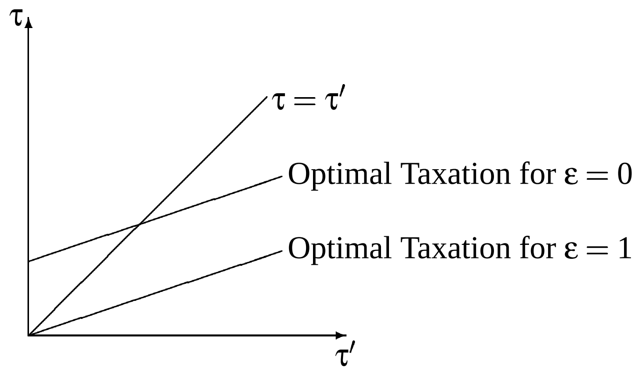

Figure 2: Optimal fiscal policy with government bonds

If there is no congestion ($\varepsilon = 1$), uniform taxation of deterministic and stochastic income ($\tau = \tau'$) implies the absence of any income taxation. This outcome reflects the result in the deterministic setting, where optimal fiscal policy in the case of a pure public good leads to complete financing of the public input via a consumption tax. But with a differentiate income tax, there is nevertheless a continuum of optimal tax policies. All fiscal policies which meet equation (41) are equivalent with respect to expected intertemporal utility. The higher the taxation of average income, the higher the taxation of random income must be. Condition (41) ensures that the distortionary impact of a positive tax rate on deterministic income components is offset by the growth enhancing insurance effect of a positive tax rate on stochastic income parts.

If instead proportional congestion arises, ($\varepsilon = 0$), the equilibrium growth rate is suboptimally high and can be reduced via income taxation. Using condition (18) leads to

$$\varepsilon = 0 \quad \Longrightarrow \quad \tau = \rho\alpha\sigma^2\tau' + \frac{G}{N\bar{y}} \quad . \tag{42}$$

A policy without taxation of stochastic income parts ($\tau' = 0$) is optimal if government expenditure is fully financed by the taxation of deterministic income. Insofar it is possible to replicate the result of the deterministic model. If instead the tax rate on stochastic income is positive, this increases the optimal tax rate on deterministic income. In particular, in the case of a uniform tax rate ($\tau = \tau'$) the corresponding deterministic setting underestimates the optimal income tax rate. The reason is that taxation of uncertain income components leads to an increase in expected growth, driving it away from the Pareto–optimal growth rate. Hence, the tax rate on sure income parts has to be higher to compensate this additional positive growth effect.

If congestion is neither absent nor complete ($0 < \varepsilon < 1$), the line of optimal tax policy is situated in between the two lines in figure 2. Again, there is a continuum of optimal fiscal policies, with positive relation between the two tax rates. The suboptimally high equilibrium growth rate is reduced via taxation of deterministic income parts and in the case of a positive tax rate on stochastic income the growth enhancing insurance effect has to be compensated additionally by a higher tax rate on expected income.

6 Summary

This paper analyzes within a dynamic macroeconomic context the growth effects of technological uncertainty and a productive governmental input which may be subject to congestion. The formal frame is a stochastic endogenous growth model with productive government spending that is financed by differentiate income– and consumption taxes.

After describing the setup of the model the decentralized equilibrium is derived. An analysis of the effects of different fiscal policies follows. Due to inelastic supply of labor, taxation of consumption does not affect economic growth. This well–known feature is unchanged by congestion or uncertainty and the consumption tax amounts to a lump–sum tax. Concerning the tax on expected income, the following result is derived: Independent of congestion or uncertainty a rise in the tax on deterministic income parts leads to a decrease in the expected growth rate and thus reflects the distortionary effects of income taxation in a deterministic context. However, the analysis of the impacts of a tax on stochastic income components is more sophisticated and the growth and welfare effects are ambiguous. Depending on the degree of risk aversion and hence on the motive for precautionary savings a higher tax on risky income may increase or decrease accumulation.

Congestion influences the marginal product of capital and the individuals have too great an incentive to accumulation. Taxation of the stochastic income parts in this context may be opposite to the income tax in a deterministic context: It may rise the sub–optimally high growth rate and diminish welfare, depending on the degree of risk aversion and of congestion. A comparison of optimal and expected growth rate leads to a relation between the optimal tax rates on deterministic and stochastic income. It is shown that for optimal fiscal policies a rise in the tax on deterministic income may be offset by a decrease or an increase in the tax rate on risky income depending on the degree of risk aversion and of congestion.

These results are contrasted with the optimal policy outcomes in case of government bonds. The assumption of a continously balanced public budget is dropped in the last section of the analysis. This changes the impact of taxation of uncertain income on growth. Because of the insurance effect a tax on stochastic income parts unambiguously increases expected growth. Nevertheless, the growth effects of taxation of expected and uncertain income components are opposite and again a continuum of optimal tax policies results.

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