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Stochastic Pollution and Environmental Care in an Endogenous Growth Model

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Abstract

This paper analyzes the impact of pollution and abatement policy within a stochastic endogenous growth model. The agents have environmental preferences, but they neglect their individual contribution to aggregate abatement. Therefore, environmental care is done by the government and financed via income taxation and government bonds. Equilibrium growth depends on environmental preferences, perception of aggregate capital and risk aversion. Environmental care as well as fiscal policy are analyzed. Due to environmental preferences and partial anticipation of the dependence between aggregate and individual capital, government debt influences equilibrium growth. Hence, income taxation has an additional indirect impact on accumulation through the simultaneous adjustment of portfolio choice. From numerical simulation it can be concluded that the optimal income tax rate decreases with the perception of the influence of individual on aggregate capital. In contrast, the impact of environmental preferences and uncertainty on optimal financing is ambiguous.

Zusammenfassung

Der vorliegende Ansatz analysiert den Einfluß von Umweltverschmutzung und Umweltschutzausgaben auf den Wachstumsprozeß im Rahmen eines stochastischen endogenen Wachstumsmodells, wobei die Umweltverschmutzung durch Güterproduktion entsteht und Nutzeneinbußen verursacht. Da die Wirtschaftssubjekte den Einfluß ihrer individuellen Umweltschutzausgaben auf die Umweltqualität vernachlässigen, wird die Aufgabe der Umweltreinhaltung durch den Staat ausgeübt, der seine Ausgaben durch Einkommensbesteuerung sowie Staatsverschuldung finanziert. Die Höhe des gleichgewichtigen Wachstums hängt von der Wahrnehmung des individuellen Einflusses auf die Umweltverschmutzung sowie den Präferenzparametern für Umwelt und Risikoneigung ab. Da die Wirtschaftssubjekte den Einfluß der individuellen Produktion auf die Umweltverschmutzung und somit auf den intertemporalen Nutzen zum Teil antizipieren, hat die Höhe der Staatsverschuldung Auswirkungen auf das Wachstum im Marktgleichgewicht. Somit gibt es neben dem direkten Wachstumseffekt der Einkommensteuer auch einen indirekten Effekt durch die Portfolioanpassung. Es resultiert ein uneindeutiger Gesamteffekt. Anhand numerischer Berechnungen wird gezeigt, daß der optimale Einkommensteuersatz abnimmt, wenn die Wahrnehmung des individuellen Einflusses auf die Umweltqualität zunimmt. Eine Zunahme der Unsicherheit oder des Disnutzens aus Umweltverschmutzung hingegen zeigen uneindeutige Effekte auf die optimale Fiskalpolitik.

1 Introduction

Recent contributions analyze pollution and environmental care within the setting of endogenous growth models. In general, pollution leads to a decline in optimal growth. Eventually, growth can even cease due to increasing environmental costs of production. There is a trade–off between consumption and pollution which is crucial for the performance of the economy: If the environmental costs become sufficiently high in the process of ongoing growth, abatement activities induce a decline in the long–run rate of return which finally can limit growth.

Particularly in the context of environmental issues, uncertainty about future consequences of present actions is of major interest. Furthermore, the impact of any environmental policy depends crucially on the existence of uncertainty. Governmental activities influence not only the expected values of economic variables but also their volatility and in general this leads to counter working effects on market equilibrium. These were first discussed by Eaton (1981) who analyzes the ambiguous impact of income taxation in a stochastic growth model with linear technology. More recent papers e. g. of Turnovsky (1993, 1995, 1999, 2000), Smith (1996), Corsetti (1997) or Clemens and Soretz (1997) extend this framework and demonstrate the role of governmental activities in various settings of stochastic endogenous growth.

The Ramsey problem of pollution in a dynamic economy is based on Forster (1973), whose framework was extended e. g. by Gruver (1976), Luptacik and Schubert (1982), Siebert (1987) or Van der Ploeg and Withagen (1991). These authors analyze the effects of pollution in neoclassical growth models. The common outcome is that pollution induces a decline in the optimal steady state capital stock. Nevertheless, the question whether environmental concerns are consistent with ongoing growth or not can only be addressed within the setting of endogenous growth. Gradus and Smulders (1993) as well as Stokey (1998) consider environmental preferences and various technologies with constant returns in the accumulable inputs to allow for ongoing growth.

Gradus and Smulders (1993) show a negative relation between optimal growth and pollution disutility in the case of constant returns to capital and with endogenous abatement activities which determine the level of pollution. In contrast, they demonstrate that environmental preferences in the framework of Lucas (1988) have no effect on long–run growth. If additionally the learning ability is reduced by pollution, abatement can even enhance growth in the case where the rise in capital productivity dominates the negative growth effect associated with the crowding out of investment. Stokey (1998) analyzes a model with linear production technology and endogenous emission standard. Pollution is assumed proportional to total output. In this setting, sustained growth is not optimal, that is, growth ceases in the long run. During transition, pollution is an asymmetric hump–shaped function of income, as empirically found by Grossman and Krueger (1995).

The model considered here focuses on the interactions between environmental preferences, risk aversion and the perception of the dependence between aggregate and individual capital. Emissions reduce individual expected utility and hence social welfare. The level of pollution depends on the relation between abatement activities and capital stock. Since there is a continuum of identical individuals and abatement is nonrival and nonexcludable, the representative agent neglects his contribution to aggregate abatement. Thus, environmental care is assigned to government and financed via income taxation and government bonds.

Furthermore, the level of emissions is perceived to depend not only on individual capital accumulation but also on aggregate capital, which seems to be exogenous to individual decisions. The extent to which the agent perceives pollution to depend on individual capital is parameterized and both polar cases are included: On the one hand the case where agents are aware of the true relation between pollution and individual capital accumulation and on the other hand the case where pollution is perceived as purely exogenous. This setting draws back on the formulation of congestion functions in the public goods literature as e. g. in Edwards (1990), Glomm and Ravikumar (1994) or Turnovsky and Fisher (1998).

Building on the assumptions of the model described in section 2, the market equilibrium is analyzed in section 3. Due to the environmental preferences capital accumulation has a secondary, welfare diminishing effect which in part — depending on the perception of capital in the pollution function — is external. Hence, in contrast to most stochastic endogenous growth models, expected growth depends on equilibrium portfolio composition and thus on government debt. The usual separability of the growth process from portfolio choice does not hold. Instead this interdependence leads to various interacting effects of fiscal policy on macroeconomic equilibrium.

Market equilibrium is contrasted with social optimum. It can be shown that in the social optimum pollution is constant and depends on the underlying production and utility parameters. Furthermore, the optimal level of emissions in the setting considered here differs from the outcome in the corresponding deterministic model. If (constant) relative risk aversion is less than unity, the deterministic model underestimates optimal pollution. If instead relative risk aversion is greater than unity, optimal emission derived under certainty exceeds the level derived with uncertainty. Only with the assumption of logarithmic utility it is possible to apply the outcome from the deterministic setting unchanged.

Section 4 analyzes the growth effects of fiscal policy. It is no longer possible to obtain a straight forward description of the impact of government activities on either equilibrium growth or social welfare. Therefore, the analysis of fiscal policy requires numerical simulation. It is shown that the optimal composition of income tax and government debt depends on the relation between relative risk aversion, perception parameter and environmental preferences. Various parameter settings are discussed and it can be concluded that the optimal share of tax financing increases with the perception of aggregate capital to be exogenous and reacts in ambiguous way with environmental preferences and risk aversion respectively.

Section 5 discusses the polar case where individuals ignore completely the influence of private capital accumulation on pollution. This setting corresponds to the deterministic model analyzed e. g. by Gradus and Smulders (1993). The interdependence between government debt and equilibrium growth vanishes in this case. Thus, a closed form solution for optimal financing can be derived, but once more, the outcomes of the corresponding deterministic model cannot be applied to the case of uncertainty, except for logarithmic preferences. If relative risk aversion is low, the optimal income tax increases with uncertainty whereas with a high degree of risk aversion, the influence is ambiguous. It is shown that optimal income taxation in this setting collects the right amount to finance abatement. Hence, government debt is zero in optimum. Finally, section 6 concludes the results.

2 The model

Pollution causes disutility and hence damages social welfare. With this formulation, the underlying paper builds on the analysis of Gradus and Smulders (1993) as well as Stokey (1998), and draws back on the earlier approaches of Luptacik and Schubert (1982) or Van der Ploeg and Withagen (1991). Consider a continuum of identical infinitely long living households who maximize expected lifetime utility

$$U = \mathrm{E}_0 \left[\int_0^\infty e^{-\rho t} u(c(t), P(t))\, dt \right] \quad . \tag{1}$$

One could think of a long–lived dynasty, because altruism between generations is very plausible in the context of environmental issues. Consumption is represented by $c(t)$ and pollution by $P(t)$. The rate of time preference $\rho > 0$ is assumed to be constant. E_0 denotes the expected value conditional on time 0 information.

Instantaneous utility exhibits constant relative risk aversion

$$u(c(t)) = \begin{cases} \dfrac{(c(t)P(t)^{-\gamma})^{1-\varepsilon}}{1-\varepsilon} & \text{for} \quad \varepsilon \neq 1 \\ \ln c(t) - \gamma \ln P(t) & \text{for} \quad \varepsilon = 1 \quad . \end{cases} \tag{2}$$

Environmental preferences come into play for $\gamma > 0$ and with higher γ disutility out of pollution gains importance. $\varepsilon > 0$ represents the degree of relative risk aversion.

The homogenous good is produced according to the linear individual stochastic production function

$$f(k(t)) = Ak(t)(dt + \sigma_y dz(t)) \quad . \tag{3}$$

The deterministic counterpart of this technology was popularized by Rebelo (1991) and the stochastic version draws back on Eaton (1981). Uncertainty enters the economy via an aggregate productivity shock $dz(t)$, which is a Wiener process with $dz \sim N(0, dt)$. A is expected capital productivity and $k(t)$ denotes a broad measure of capital available to the representative firm.

Pollution depends on the relation between capital and abatement activities, and it is modeled as a flow variable. This formulation recurs on Gradus and Smulders (1993). In e. g. Forster (1973), Van der Ploeg and Withagen (1991), Stokey (1998) or Jones and Manuelli (2001) output is the source of pollution and in e. g. Luptacik and Schubert (1982) as well as Van der Ploeg and Withagen (1991) pollution as a stock variable is considered. It can be shown that under the assumptions further taken here, the outcomes remain the same with a stock of pollution and are independent of the source of environmental degradation.

In this paper, the approach is extended and the perception of the capital stock is taken into account. Pollution, $P(t)$, is determined by the relation of aggregate capital, $K(t)$, individual capital, $k(t)$, and abatement effort, $E(t)$

$$P(t) = \left(\frac{K(t)^{\delta} k(t)^{1-\delta}}{E(t)} \right)^{\alpha} dt \quad \alpha, \delta \in [0,1] \quad . \tag{4}$$

The extent to which the agent perceives pollution to depend on exogenous aggregate capital is parameterized by δ. In equilibrium, aggregate capital equals individual capital, because all households are identical and the population size is normalized to unity. Thus, $\delta = 0$ represents perfect knowledge about the effect of individual capital accumulation on pollution. On the other hand, $\delta = 1$ is associated with the situation where the representative agent neglects completely his individual contribution to aggregate capital. Pollution is perceived not to depend in any way on individual accumulation. For $0 < \delta < 1$ perception of capital is in between these polar cases. Individual capital is expected to influence pollution, but the extent of the dependence is underestimated.

Pollution control, $E(t)$, is assumed to be nonrival and nonexcludable. Thus, with a continuum of households, the effect of individual effort on environmental quality vanishes. Hence, abatement activity is done by the government and financed by income taxation and government debt. The government levies a proportional income tax at rate τ.

With stochastic production, tax revenues out of income taxation are stochastic whereas expenditures for abatement activities are deterministic. Hence, the government budget cannot be balanced in each instant of time. Instead it is closed by issuing bonds with stochastic value $B(t)$. These bonds are assumed to be perpetuities which pay an after tax expected interest rate i as well as a net stochastic interest rate dz_i. Both parts of the return on government bonds have to be determined in equilibrium. The value of government bonds evolves according to

$$dB(t) = [E(t) + iB(t) - \tau AK(t)]dt + B(t)\,dz_i - \tau AK(t)dz \quad . \tag{5}$$

In the following section, individual optimization and the resulting macroeconomic equilibrium will be analyzed.

3 Market equilibrium

In the presence of government bonds, the representative individual has to choose not only optimal capital accumulation but also the utility maximizing portfolio. Individual wealth,

$w(t)$, is composed of physical capital, $k(t)$, and financial wealth, $b(t)$. The portfolio share of capital is denoted by n. Thus, the individual wealth restriction is

$$dw = [(1-\tau)Anw + i(1-n)w - c]dt + (1-\tau)Anw\sigma_y\,dz + (1-n)w\,dz_i \tag{6}$$

and the variance of wealth (divided through dt) evolves to

$$\sigma_w^2 = (1-\tau)^2A^2n^2w^2\sigma_y^2 + 2(1-\tau)An(1-n)w^2\sigma_{yi} + (1-n)^2w^2\sigma_i^2 \tag{7}$$

where σ_{yi} denotes the covariance between output and return on bonds and σ_i^2 the variance of the return on bonds. These both depend on the endogenous stochastic process of the return on bonds, dz_i, which has to be determined in equilibrium.

Expected intertemporal utility (1) is maximized with respect to the wealth constraint (6) while tax rates as well as initial values k_0 and z_0 are given. With the specification for utility given above, the maximized lifetime utility can be shown to be of the time–separable form $e^{-\rho t}J(w(t))$. Employing Itô's Lemma the stochastic Bellman equation is given by

$$\mathcal{B} = e^{-\rho t}u(c,P) - \rho e^{-\rho t}J(w) + e^{-\rho t}J'(w)\frac{\mathrm{E}[dw]}{dt} + \frac{1}{2}e^{-\rho t}J''(w)\,\sigma_w^2 \quad . \tag{8}$$

Optimization of the representative individual requires the expected utility maximizing choice of consumption and portfolio. There is no individual choice about the level of abatement activities, since the individual influence on environmental restoration is neglected. Hence, maximization leads to the first order conditions with respect to consumption, c, and portfolio share of capital, n,

$$c^{-\varepsilon}P^{-\gamma(1-\varepsilon)} = J'(w) \tag{9}$$

$$\frac{\alpha\gamma(1-\delta)}{n}c^{1-\varepsilon}P^{-\gamma(1-\varepsilon)} + J'(w)\,[(1-\tau)Aw - iw] + \frac{1}{2}J'''(w)\frac{\partial\sigma_w^2}{\partial n} = 0 \quad . \tag{10}$$

Equation (9) assures the equality of marginal utility out of consumption across time. For a setting which satisfies the following conditions: (i) time invariant relative risk aversion, (ii) constant marginal product of capital, (iii) variance proportional to the square of the state variable Malliaris and Brock (1982, p. 178) show that there exists a closed form solution. In the setting considered here, these conditions are met as can be seen from equations (2), (3) and (7). The value function then has the same shape as the instantaneous utility function. This outcome can be demonstrated by the conjecture of a constant relation between consumption and wealth, which is denoted by μ. Hence, the value function is given by

$$J(w) = \begin{cases} \mu^{-\varepsilon} \dfrac{(wP^{-\gamma})^{1-\varepsilon}}{1-\varepsilon} & \text{for} \quad \varepsilon \neq 1 \\ \mu^{-1} \ln(wP^{-\gamma}) & \text{for} \quad \varepsilon = 1 \end{cases} \quad . \tag{11}$$

Equation (10) determines the optimal portfolio. In an equilibrium with constant portfolio shares all assets grow with the same stochastic rates

$$\frac{dw}{w} = \frac{dk}{k} = \frac{db}{b} \quad . \tag{12}$$

That is, expected growth rates are equal and the stochastic processes evolve in the same way. In particular, equating the stochastic components of (12) leads to the equilibrium stochastic process of bond return

$$dz_i = \frac{1-(1-\tau)n}{1-n} A\sigma_y \, dz \tag{13}$$

which is proportional to the productivity shock. Together with the value function (11) and condition (10) it is now possible to derive the expected rate of return on bonds[1]

$$i = (1-\tau)A + \varepsilon \frac{A^2\sigma_y^2\tau}{1-n} - \alpha\gamma(1-\delta)\mu^{\varepsilon} \quad . \tag{14}$$

For optimal capital accumulation the Bellman equation is equal to zero. This leads to the consumption–wealth ratio

$$\mu = \frac{\rho + (\varepsilon-1)(1-\tau)A - \frac{1}{2}\varepsilon(\varepsilon-1)(1-2\tau)A^2\sigma_y^2}{\varepsilon - \alpha\gamma(1-\delta)\frac{1-\varepsilon(1-n)}{n}} \tag{15}$$

which indeed is constant and confirms the conjecture (11) of the value function as long as portfolio composition is constant over time. The propensity to consume out of wealth depends on the underlying parameters as well as on the fiscal instruments. Two special cases can be noted here: If aggregate capital is perceived to be completely exogenous to the accumulation decision of the individual, $\delta = 1$, or if the environmental preferences

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[1] Equations (13) and (14) are only valid in the case $n \neq 1$. Otherwise, there is no government debt and therefore the government budget is balanced in each instant of time. This will only be a feasible equilibrium if the taxation of stochastic income parts vanishes, that is τ is only applied to expected income. Government bonds then are a sure asset ($dz_i = 0$) and the resulting rate of return is $i = (1-\tau)A - \varepsilon A^2\sigma_y^2$.

vanish, $\gamma = 0$, the second term in the denominator is zero and the consumption–wealth ratio corresponds to the linear model without pollution. In the first case the individuals aren't aware of their influence on disutility out of pollution. In the second case there is no negative impact of pollution on utility.

Furthermore, the transversality condition

$$\lim_{t\to\infty} \mathrm{E}\,\left[e^{-\rho t} J(w)\right] = 0 \tag{16}$$

must be satisfied for expected utility to be bounded and in order to assure feasible consumption paths. As Merton (1969) first demonstrated, the transversality condition is in this setting equivalent to the condition for a positive ratio between consumption and wealth.

The expected growth rate of the economy, φ, can be obtained from the individual wealth constraint (6)

$$\varphi \equiv \frac{\mathrm{E}[dw]}{w\,dt} = \frac{1}{\tilde{\varepsilon}}\left((1-\varepsilon+\tilde{\varepsilon})(1-\tau)A-\rho\right)+\frac{\varepsilon}{2\tilde{\varepsilon}}(\varepsilon-1+2\tau(1-\varepsilon+\tilde{\varepsilon}))A^2\sigma_y^2 \tag{17}$$

with $\tilde{\varepsilon}$ defined as follows

$$\tilde{\varepsilon} \equiv \frac{\varepsilon(n+\alpha\gamma(1-\delta)(1-n))-\alpha\gamma(1-\delta)}{n+\alpha\gamma(1-\delta)(1-n)} \leq \varepsilon \quad . \tag{18}$$

In addition to the usual utility and production parameters, equilibrium expected growth is influenced by environmental preferences, γ, perception of aggregate capital, δ, and the volatility of the technological disturbance, σ. Each of the relations is ambiguous and depends on the level of the certainty equivalent of portfolio return and the degree of relative risk aversion, respectively. A detailed discussion will be given below. To ensure feasible solutions, $\tilde{\varepsilon}$ is assumed to be positive for all values of relative risk aversion ε. Therefore, parameters have to fulfill the condition $1-\alpha\gamma(1-\delta) > 0$ that is, environmental preferences shouldn't be too strong.

The first term of the expected growth rate (17) equals the growth rate of the corresponding deterministic model. The second term reflects the response of the risk averse representative individual to uncertainty in future income flows. Due to pollution which is positively related to the capital stock, there is a negative external effect of capital accumulation.[2]

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[2] In the case of perfect anticipation $\delta = 0$, this additional negative effect of capital accumulation is completely internalized within the individual intertemporal decision.

This has an ambiguous effect on expected growth. Since the average capital return is overestimated, growth tends to be suboptimally high without taxation. In contrast, the overestimation of the risk associated with capital returns ceteris paribus leads to suboptimally low accumulation. As long as the certainty equivalent of portfolio return[3] is positive, the first effect dominates and the negative externality of capital accumulation leads to suboptimally high growth.

Perception of aggregate capital as well as environmental preferences influence expected growth in an ambiguous way

$$\frac{\partial\varphi}{\partial\delta} = \frac{n\alpha\gamma}{\tilde{\varepsilon}}((1-\tau)A(1-\varepsilon A\sigma_y^2)-\varphi) \tag{19}$$

$$\frac{\partial\varphi}{\partial\gamma} = -\frac{n(1-\delta)}{\tilde{\varepsilon}}((1-\tau)A(1-\varepsilon A\sigma_y^2)-\varphi) \quad . \tag{20}$$

Both effects depend on the relation between expected growth and the certainty equivalent of portfolio return. As long as the growth rate falls short of the certainty equivalent, the growth rate increases with the parameter δ indicating the degree to which aggregate capital is perceived to be exogenous. With smaller δ the individuals perceive a stronger influence of individual on aggregate capital. That is, they anticipate a greater effect of capital accumulation on pollution and on disutility out of pollution. This leads to a decrease in equilibrium accumulation. Thus, a smaller value of δ induces less growth and vice versa.

With respect to environmental preferences, the impact is similar. As long as the growth rate is smaller than the certainty equivalent, increasing importance of pollution for utility (that is a higher γ) leads to a reduction in accumulation. The negative effect of capital accumulation is given more weight in optimization and hence savings are diminished.

If individuals ignore their private contribution to aggregate capital completely ($\delta = 1$), the direct impact of environmental issues on decentralized growth vanishes. Nevertheless, the government has to take environmental degradation as well as abatement activities into account in order to set the fiscal policy parameters optimally. The tax rate on deterministic income components as well as the taxation of uncertain income parts affect savings. Thus, accumulation is adjusted optimally through income taxation. More simply, optimal

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[3] The certainty equivalent of portfolio return can be evaluated according to Merton (1992, p. 45) and is positive if $1-\varepsilon A\sigma_y^2 > 0$.

taxation balances marginal expected utility out of consumption for marginal expected disutility out of pollution. The effects are discussed in detail in section 5.

In general, uncertainty has a positive income and a negative substitution effect on savings: The income effect refers to the diminished expected utility out of future income streams due to increasing risk. Thus, there is an incentive to enhance savings in order to compensate for this impact and to equalize expected marginal utility across time. The substitution effect reflects the incentive to decrease savings because in the presence of higher risk accumulation is less attractive for risk averse individuals. In the model considered here, the effect of uncertainty on growth

$$\frac{\partial \varphi}{\partial \sigma_y^2} = \frac{\varepsilon}{2\tilde{\varepsilon}}(\varepsilon - 1 + 2\tau(1 - \varepsilon + \tilde{\varepsilon}))A^2 \tag{21}$$

can be shown to depend in a simple way on the level of relative risk aversion, since the term

$$1 - \varepsilon + \tilde{\varepsilon} = \frac{n(1 - \alpha\gamma(1 - \delta))}{n(1 - \alpha\gamma(1 - \delta) + \alpha\gamma(1 - \delta)} \quad \Longrightarrow \quad 0 < 1 - \varepsilon + \tilde{\varepsilon} < 1 \tag{22}$$

is independent of relative risk aversion and within the unity interval. With a relative risk aversion sufficiently high, the income effect prevails and there is a motive for precautionary savings (see Leland (1968) or Sandmo (1970)). In this case uncertainty leads to higher savings. Vice versa, if the coefficient of relative risk aversion is sufficiently low, an increase in uncertainty induces a reduction in optimal accumulation.

The productivity shock is assumed to be a Wiener process. Hence, time t capital is a geometric Wiener process and is lognormally distributed. Given the initial values of capital k_0 and the stochastic process z_0 at time 0, capital evolves according to

$$k(t) = k_0 \, e^{\left(\varphi - \frac{1}{2}\alpha^2\sigma^2\right)t + \alpha\sigma[z(t) - z_0]} \quad . \tag{23}$$

Maximal expected lifetime utility is then given by

$$U = \frac{(A - \varphi - \eta)^{1-\varepsilon}\eta^{\alpha\gamma(1-\varepsilon)}k_0^{1-\varepsilon}}{(1 - \varepsilon)(\rho - (1 - \varepsilon)(\varphi - \frac{1}{2}\varepsilon A^2\sigma_y^2))} \tag{24}$$

where the relation between abatement and physical capital, which is set by the government and will be determined in the next section, is denoted by $\eta = E/k$ and market clearing requires $c/k = A - \varphi - \eta$.

4 Fiscal and environmental policy

This section builds on the determination of the dynamic equilibrium and analyzes the impact of income taxation and government debt on steady state growth. Afterwards, conditions for optimal environmental policy are stated.

The ambiguous effects of fiscal policy can be assigned to the counteracting impact of fiscal policy on expected growth. There are both a direct and an indirect growth effect of income taxation

$$\frac{d\varphi}{d\tau} = \underbrace{\frac{\partial\varphi}{\partial\tau}}_{\text{direct effect}} + \underbrace{\frac{\partial\varphi}{\partial n}\frac{\partial n}{\partial\tau}}_{\text{indirect effect}} \quad . \tag{25}$$

The direct impact of a change in the income tax rate is the well known ambiguous growth effect of income taxation in a stochastic growth model

$$\frac{\partial\varphi}{\partial\tau} = -\frac{1-\varepsilon+\tilde{\varepsilon}}{\tilde{\varepsilon}} A(1-\varepsilon A\sigma_y^2) \quad . \tag{26}$$

It can be decomposed into a growth diminishing distortionary effect which is associated with the reduction in expected capital return and a growth enhancing insurance effect which is associated with the decline in capital risk. For a detailed discussion of the effects of taxation within stochastic models of endogenous growth see e. g. Eaton (1981), Turnovsky (1995), Smith (1996), Corsetti (1997) or Clemens and Soretz (1997). The direct effect of taxation on growth can be shown to be negative if and only if the certainty equivalent of the portfolio return is positive. This condition can be interpreted in the following way: With a positive certainty equivalent risk does not dominate the model. The technology is "certain enough" as to assure that the effects of the underlying deterministic structure prevail. A negative certainty equivalent describes a situation where the uncertain capital income flow yields the same utility as a certain interest rate which is negative. The following analysis will be restricted to parameter settings which assure a positive certainty equivalent, because this situation fits better to reality.

But furthermore, in this model there is an indirect impact of income taxation on growth. Any change in the income tax rate leads to a simultaneous adjustment of portfolio choice and government debt, respectively. Since with the underlying environmental preferences expected growth depends on the portfolio share of capital, the adjustment of portfolio choice induces an ambiguous indirect growth effect. The equilibrium portfolio composition can by evaluated as

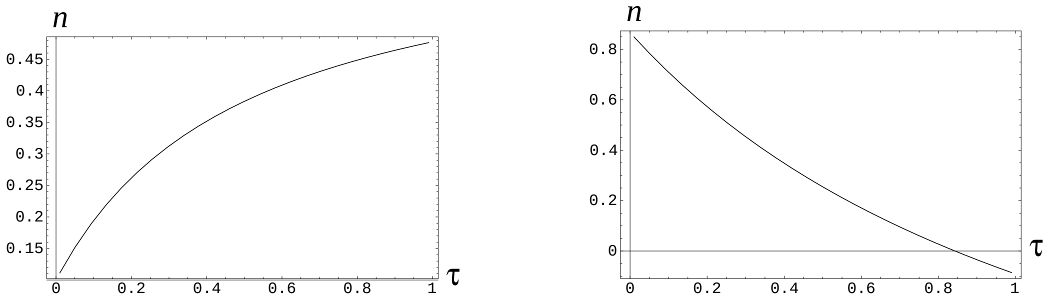
$$n = \frac{\rho(1-\vartheta) - \vartheta(\varepsilon-1)(A-\eta) + (\varepsilon-1)(1-\tau)A - \frac{\varepsilon}{2}(\varepsilon-1)(1+\vartheta-2\tau)A^2\sigma_y^2}{(1-\vartheta)(\varepsilon(A-\eta) - (1-\tau)A + \rho - \frac{\varepsilon}{2}(\varepsilon-1+2\tau)A^2\sigma_y^2)} \tag{27}$$

where η denotes the relation between abatement and capital stock, E/k, and $\vartheta = \alpha\gamma(1-\delta)$ is defined for notational convenience.

Pollution is positively related to the capital stock. Hence, an increase in the portfolio share of capital ceteris paribus aggravates pollution. This induces a decline in optimal accumulation as long as the certainty equivalent of portfolio return is positive

$$\frac{\partial\varphi}{\partial n} = \frac{1}{\tilde{\varepsilon}}((1-\tau)A(1-\varepsilon A\sigma_y^2))\frac{(1-\vartheta)\vartheta}{(1+\vartheta(1-n))^2} \quad . \tag{28}$$

Nevertheless, the effect of income taxation on portfolio composition is ambiguous. On the one hand expected net capital return decreases with a rise in the tax rate. Thus, physical capital gets less attractive compared with government bonds. On the other hand the volatility of capital return diminishes with increasing taxation whereas the uncertainty of bond return rises.



(a) *Relative risk aversion* $\varepsilon = 0.5$. (b) *Relative risk aversion* $\varepsilon = 3$.

Figure 1: *Portfolio response on tax increase for small versus high relative risk aversion.*

The effect of income taxation on portfolio composition is illustrated in figure 1. The parameters are set in the following way: $\rho = 0.03$, $\gamma = 1$, $A = 0.4$, $\sigma_y = 0.01$, $\delta = 0.5$,

$\alpha = 0.5$. η is evaluated according to equation (31) developed below at the optimal level. For small values of relative risk aversion ε the derivative of the portfolio share n with respect to τ can be shown to be positive, whereas with a relative risk aversion sufficiently high the derivative becomes negative. In the first case, the positive effect of taxation on the portfolio share of capital dominates: The increase in the tax rate induces a shift towards physical capital. Contrary, if the agents are sufficiently risk averse, the negative effect of taxation on the portfolio share of capital dominates. In this case, the decrease in expected net capital return induces a decrease in capital demand. The optimal portfolio share of physical capital is reduced. Subfigure 1(b) additionally shows that there is an upper bound to the income tax rate which must be kept in order to assure feasible solutions with positive values for capital.

In the model considered here, optimal fiscal and environmental policy can be decomposed into two steps: First, the government has to choose the optimal level of abatement expenditures. Second, optimal financing has to be analyzed. Referring to optimal pollution control, expected lifetime utility (24) is maximized with respect to the environmental expenditure rate, η, and with respect to the growth rate, φ,

$$\frac{\partial U}{\partial \eta} \overset{!}{=} 0 \iff \eta^* = \frac{\alpha\gamma}{1+\alpha\gamma}(A-\varphi) \tag{29}$$

$$\frac{\partial U}{\partial \varphi} \overset{!}{=} 0 \iff \varphi^* = \frac{1}{\varepsilon}(A-\eta-\rho) + \frac{\varepsilon-1}{2}A^2\sigma_y^2 \quad . \tag{30}$$

Combination of these two optimality conditions leads to

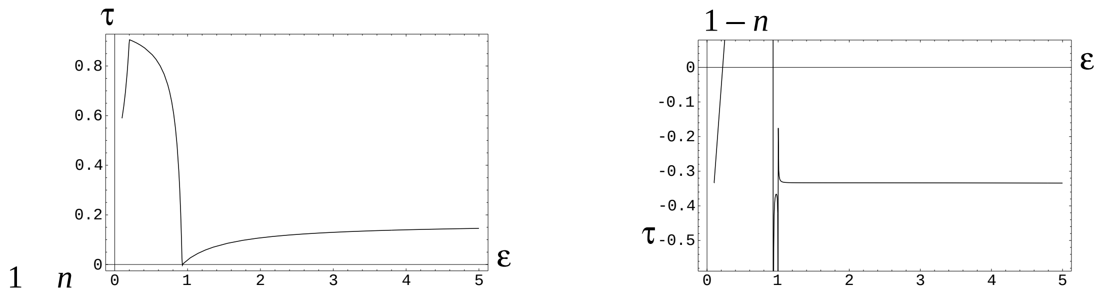
$$\eta^* = \frac{\alpha\gamma}{\varepsilon(1+\alpha\gamma)-\alpha\gamma}(\rho+(\varepsilon-1)A) + \varepsilon\frac{1-\varepsilon}{2}A^2\sigma_y^2 \tag{31}$$

$$\varphi^* = \frac{1+\alpha\gamma}{\varepsilon(1+\alpha\gamma)-\alpha\gamma}\left(\frac{A}{1+\alpha\gamma} - \rho + \varepsilon\frac{\varepsilon-1}{2}A^2\sigma_y^2\right) \quad . \tag{32}$$

The optimal rate of abatement activities is given by equation (31) and differs with respect to the second term from the corresponding deterministic model. That is, optimal environmental care increases (decreases) with uncertainty if risk aversion is less (higher) than unity. Hence, in general the outcome of the deterministic model doesn't apply to the case of uncertainty. If risk aversion is sufficiently low ($\varepsilon < 1$), optimal pollution control is underestimated by the setting without risk (and vice versa). Only with logarithmic preferences ($\varepsilon = 1$) optimal environmental care is independent of uncertainty. In this case, income and substitution effect of risk on environmental care balance.

The impact of environmental preferences on optimal expected growth (32) is ambiguous. The intertemporal elasticity of substitution as well as capital productivity are influenced by the disutility out of pollution. Within social optimization the negative external effect of capital accumulation is taken into account. Perception of the capital stock is equivalent to real capital stock, that is $\delta = 0$ in the determination of pollution (4).

Second, the government solves for the optimal financing of environmental expenditures. Optimal financing requires the identity of decentralized expected growth (17) and optimal expected growth (32). Since income taxation has various and ambiguous impacts on expected growth, optimal financing will only be illustrated numerically. Using the portfolio share of capital (27) as well as optimal environmental care (31) to determine the decentralized growth rate (17) and equating decentralized and optimal growth leads to an optimal tax rate as given subfigure 2(a). Government debt results residually and is illustrated in subfigure 2(b). The parameter settings are the same as in figure 1.

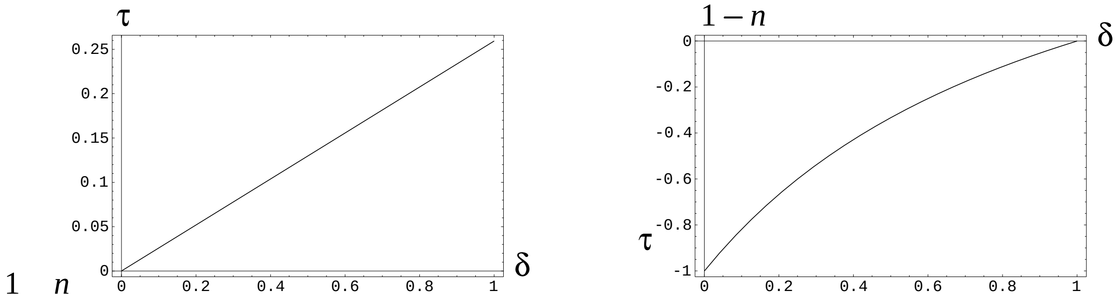


(a) *Optimal tax rate.*

(b) *Optimal government debt.*

Figure 2: *Relative risk aversion and optimal financing of environmental care.*

It can be seen that optimal financing depends crucially on the degree of relative risk aversion. As long as relative risk aversion is less (higher) than unity, the optimal income tax decreases (increases) with an increase in ε. Subfigure 2(b) shows that the optimal portfolio share of government bonds is negative if the degree of relative risk aversion is above unity. Thus, the equilibrium value of government bonds is negative for optimal financing and government results to be a net creditor to the public. Optimal expenditures for environmental care are higher than optimal tax revenues. The negative externality of capital accumulation allows for positive income taxation in order to diminish equilibrium growth, but the revenue out of income taxation is not sufficient for optimal environmental care. Government budget is closed via interest payments of the private households.



(a) *Optimal tax rate.*

(b) *Optimal government debt.*

Figure 3: *Perception of aggregate capital and optimal financing of environmental care.*

Figure 3 shows the influence of the perception of capital on the optimal financing of abatement activities. Additionally to the previous figures, the degree of relative risk aversion is set to $\varepsilon = 3$. An increase in the perception parameter δ corresponds to a decline in the anticipation of the interdependence between individual and aggregate capital. Hence, with increasing δ agents neglect a greater part of their contribution to pollution. The negative externality of capital accumulation rises with δ and allows for an increase in optimal income taxation. Hence, the optimal value of government bonds is still negative but the absolute value diminishes.

5 A special case: Agents completely neglect their contribution to aggregate capital

In this section I demonstrate the polar case where individuals completely ignore their influence on aggregate capital and hence on pollution. This corresponds to the setting $\delta = 1$. Since the representative agent doesn't take the influence of capital accumulation on pollution into account, portfolio composition doesn't have welfare effects in this situation.

Hence, equilibrium growth is independent of the portfolio share of physical capital

$$\varphi_1 = \frac{1}{\varepsilon}((1-\tau)A-\rho)+\frac{1}{2}(\varepsilon-1+2\tau)A^2\sigma_y^2 \quad . \tag{33}$$

In this setting the environment has no direct impact on equilibrium growth. Agents ignore their individual influence on aggregate capital as well as on aggregate abatement expenditures. Thus, pollution is perceived to be purely exogenous and doesn't have any consequences for individual optimization.

The optimal ratio of abatement effort and capital η^* is independent of the perception of capital and still given by equation (31). But now it is possible to evaluate a closed form solution for optimal income taxation. Equating decentralized growth φ_1 and optimal growth φ^* now leads to

$$\tau^* = \frac{\alpha\gamma}{A(\varepsilon+\alpha\gamma(1-\varepsilon))(1-\varepsilon A\sigma_y^2)}\left(\rho+(\varepsilon-1)A-\varepsilon\frac{\varepsilon-1}{2}A^2\sigma_y^2\right) \quad . \tag{34}$$

The effect of uncertainty on the optimal tax rate is ambiguous and depends on the degree of risk aversion. If relative risk aversion is higher than unity ($\varepsilon > 1$), there is a motive for precautionary savings. The reduction of risk associated with the taxation of stochastic income parts discourages accumulation and encourages consumption. Thus, the growth diminishing effect of taxation of deterministic income components (due to a reduction in expected capital return) is reinforced by the negative growth effect of taxation of uncertain income parts. For this reason, the optimal tax rate tends to decrease with risk in a setting with strong risk aversion. Nevertheless, there are ambiguous additional effects through government debt and portfolio decision. Thus, the overall effect of uncertainty on optimal taxation is ambiguous.

If instead relative risk aversion is less than unity, the agents don't have a motive for precautionary savings. The substitution effect of uncertainty on savings dominates and the individuals reduce savings when uncertainty arises. Taxation of stochastic income parts reduces the volatility of capital return and thus induces a decrease in the risk associated with capital accumulation. With a relative risk aversion less than unity individuals now increase savings because due to the decrease in risk capital accumulation gets more attractive. Hence, the growth effects of income taxation of deterministic and stochastic income components are counter working and the overall growth effect of income taxation is less than under certainty.[4] This leads immediately to an optimal income tax rate which is increased by uncertainty.

Furthermore, it is possible to evaluate the equilibrium portfolio share of physical capital

$$n_1 = \frac{\rho + (\varepsilon - 1)\left(\varphi_1 - \frac{1}{2}\varepsilon A^2 \sigma_y^2\right)}{A - \eta - \varphi_1} \quad . \tag{35}$$

Tax policy influences expected growth and thus portfolio composition. It can be shown that the portfolio share diminishes with an increase in the income tax rate

$$\frac{\partial n_1}{\partial \tau} = -\frac{A(1 - \varepsilon A \sigma_y^2)}{A - \eta - \varphi_1} < 0 \quad . \tag{36}$$

Taxation of capital return reduces the incentive for capital accumulation and induces a shift towards government bonds. The tax revenues rise and induce ceteris paribus (with constant environmental expenditure) an increase in the value of government bonds.

For optimal environmental care, η^*, and optimal income tax, τ^*, equation (35) leads to a portfolio share of $n_1 = 1$. That is, the abatement expenditures are completely financed by the income tax. With this respect, the outcomes of the deterministic setting continue to hold. The optimal tax rate which completely internalizes the external effect at the same time collects the right amount to finance abatement. Thus, there is no need for further financing via growth neutral government debt.

The outcomes of this polar case ($\delta = 1$) are summarized in figure 4. The line with negative slope (as developed in equation (36)) identifies the correspondence between the two financing instruments government debt and income tax given in equation (35). Hence, this line indicates all feasible fiscal policies. Optimal tax policy is given by τ^* and $n = 1$. If the tax rate is less than τ^*, the portfolio share of capital exceeds unity. This situation corresponds to the case with negative value of government bonds. Government becomes a net creditor to the public.

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[4] As long as the certainty equivalent of portfolio return is positive, the growth diminishing effect of the taxation of deterministic income parts dominates. As noted above, I restrict to parameter settings which assure a positive certainty equivalent.

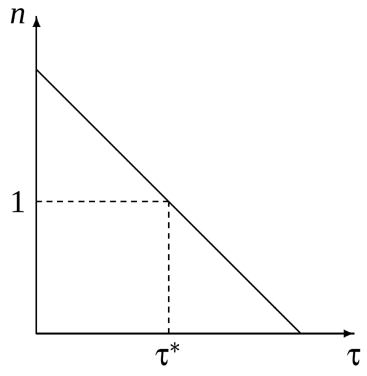

Figure 4: Optimal fiscal policy for $\delta = 1$

6 Conclusion

In this paper a stochastic endogenous growth model with pollution is analyzed. Pollution causes disutility and depends on the ratio between abatement activities and the capital stock. Since individuals neglect their contribution to aggregate environmental expenditures, these are financed by income taxation and government debt. Furthermore, the agents only partially anticipate their influence on aggregate capital. Perception of aggregate capital is parameterized and both polar cases are included: The setting where individuals completely ignore the relation between individual and aggregate capital accumulation and the contrary case with full information about this relation.

Equilibrium growth is analyzed and it is shown that environmental preferences reduce growth if and only if the certainty equivalent of portfolio return falls short of expected growth. The effect of uncertainty on growth depends on relative risk aversion. If there is a motive for precautionary savings, expected growth increases with uncertainty. Furthermore, expected growth depends on the individual perception of aggregate capital. The impact again is ambiguous and depends on the relation between certainty equivalent of portfolio return and expected growth rate.

Fiscal policy as well as optimal environmental care are analyzed. Optimal abatement activities are shown to depend on risk in an ambiguous way. If relative risk aversion is sufficiently high (low), the optimal environmental expenditure rate increases (decreases) with uncertainty. Thus, the outcomes of the deterministic setting in general don’t apply to the case of uncertainty. Only if the degree of relative risk aversion is unity, the results remain unchanged.

As long as the agents perceive a dependence between individual and aggregate capital, there is an additional negative effect of capital accumulation on intertemporal utility. This leads to an interdependence between government debt and expected growth. Hence, in contrast to most endogenous growth models, income taxation not only has a direct effect on expected growth, but it also influences portfolio choice and this leads to an additional indirect impact on the growth rate. Due to the different counter working growth effects of income taxation, numerical simulation of optimal fiscal policy is required. It is shown that the optimal income tax rate depends on the relation between relative risk aversion, perception of capital and environmental preferences.

In the last section, the outcomes are evaluated in the special case where the agents perceive aggregate capital as completely exogenous. This polar case is emphasized, because most deterministic endogenous growth models with pollution analyze this setting. It is shown that the portfolio share of physical capital decreases with the income tax rate and that optimal taxation of environmental expenditures in this case requires complete income tax financing.

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